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# Strategies for adding features to CAD models in order to optimize performance

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## Abstract

This paper presents an approach which enables new parameters to be added to a CAD model for optimization purposes. It aims to remove a common roadblock to CAD based optimization, where the parameterization of the model does not offer the shape sufficient flexibility for a truly optimized shape to be created. A technique has been developed which uses adjoint based sensitivity maps to predict the sensitivity of performance to the addition to a model of four different feature types, allowing the feature providing the greatest benefit to be selected. The optimum position to add the feature is also discussed. It is anticipated that the approach could be used to iteratively add features to a model, providing greater flexibility to the shape of the model, and allowing the newly-added parameters to be used as design variables in a subsequent shape optimization.

## Keywords

*Shape optimization, topology optimization, CAD, adjoint method*

## 1. Introduction

The goal of this work is to allow feedback from CAE simulation results to be used to optimize CAD geometry within an integrated design environment. The ability to successfully optimize a model within a design workflow depends on how the model is parameterized.

Samareh (1999) provides a survey of parameterization techniques. Mesh based parameterization does not use CAD geometry to implement a change in shape, rather modifying the analysis mesh directly. Some such approaches perturb the positions of each node independently, with the consequence that a smooth component boundary may not result (Choi and Chang 1994). Mesh morphing (Liu and Yang 2007) tools are now available in many CAE packages. These aim to parameterize the mesh, while constraining the positions of the nodes so that sharp features do not occur. Although these mesh based approaches eliminate the

need for repeatedly meshing new design iterations, the approaches require a constant mesh structure throughout the optimization, which constrains the magnitude of feasible shape changes. For a complex mesh a large number of design variables can result which can cause difficulties (Haftka and Gurdal 1992). There is also the key difficulty that when an optimized shape has been determined the creation of a representative geometric CAD model from the mesh is not trivial. A number of geometry based parameterization approaches are available which can achieve large geometry changes, making them suitable for the conceptual design effort. The discrete approach uses the coordinates of the boundary points as design variables, as shown by Campbell (1992). Braibant and Fleury (1984) note that this approach could also result in non-smooth boundaries, which can be expensive to manufacture, and instead they propose the use of the control points for the faces of boundary representation models (B-Rep) (Braid, 1975) as design variables.

For a fully integrated design process, managed in a modern product lifecycle management (PLM) system, optimized components need to be represented by the feature based CAD models which are required for downstream design and manufacturing processes. Therefore using the parameters associated with the CAD features for optimization is advantageous because the optimized model is still available as a feature based model. However the features which make up the CAD model, and the parameters which define them, limit its design space. During optimization the component can take no other shape than what is possible using the features and parameters from which it is composed. One difficulty with using the parameterization of a feature based CAD system for optimization is that the calculation of sensitivity derivatives is not easily achieved (Townsend *et al.* 1998). Lindby and Santos (1997) present an approach where the displacement of the boundary finite element mesh caused by a CAD design parameter is used to provide sensitivity values.

Armstrong *et al.* (2009) proposed the concept of parametric effectiveness as a method of rating the parameterization of a CAD model for use in optimization. Parametric effectiveness compares the performance improvement achieved when the optimum combinations of parametric changes are used to change the shape of the CAD model, to the change in performance which would be achieved if the model boundary was able to move in an arbitrary manner. This rating allows the

designer to determine how good the optimization of the CAD model will be if the parameters in the model are used as the design variables. A parametric effectiveness close to 1 suggests the parameterization of the CAD model will allow the shape to transform in a way that is close to optimum. A low parametric effectiveness suggests that the parameterization of the CAD model cannot be used to transform the shape effectively, and that more flexibility is needed in the model.

This paper addresses the question of how to proceed when the CAD model in its current form has been optimized with respect to the current set of feature parameters, and the parametric effectiveness of the model is low, so that in order for further performance improvement to be realized new features need to be added to the model. Weiss (2010) poses the question of whether structural feature creation for optimization in CAD is possible, and this paper attempts to contribute to this goal. Due to the nature of the features available in a modern CAD system, feature addition may be considered as either shape optimization or topology optimization, depending on the feature added.

Shape optimization is where the shape of the boundary of the part is changed in order to improve performance. Surveys of different approaches are provided by Haftka and Grandhi (1986) and Ding (1986). In this work shape optimization is implemented by adding a feature where the genus of the component stays constant. Topology optimization, reviewed by Eschenauer and Olhoff (2001), occurs where the addition of a feature will cause the genus of the model to change by inserting a hole or void into the domain.

Weiss (2010) presented an approach for adding extra design parameters to a feature based CAD model through the addition of extra control points for spline curves in the model. The approach proposed there is in some respects similar to the bubble method demonstrated by Eschenauer *et al.* (1994), where new “bubble” features were added to a 2D domain, and then the position and shape of the new bubble feature were optimized. This paper provides a framework within which this approach could be extended to the addition of an edge stiffener, a boss, a pocket and a hole. The sensitivity of performance to the addition of a new feature is derived, with a procedure to determine *a priori* where the new feature should be located to give the greatest improvement in performance. Once the feature is added, its parameters can be used to optimize the part.

## 2. Problem definition

### Adjoint method

In this paper the change in performance due to adding a new feature to the model is computed based on the adjoint method, an introduction to which is provided by Giles and Pierce (2000). Adjoint methods are advantageous in optimization where there is a large number of design variables, but a small number of loadcases. Once the adjoint analysis has been completed, the change in performance due to any number of infinitesimal boundary movements can be predicted without further computational analysis. For problems where there are a small number of design variables but a large number of loadcases the adjoint method is not efficient.

Typically the adjoint solution requires one further adjoint analysis after the standard analysis is complete. In this paper the adjoint method allows the change in performance to be predicted based on the change in the shape of the model which results from adding the new features, meaning a new analysis is not required to evaluate each one.

During optimization the goal is to improve the objective function,  $J$ , by changing the optimization variables. The optimization objective in this work is focused on the minimization of linear elastic strain energy in the model, which is equivalent to compliance minimization. Strain energy minimization has the advantage that it is self-adjoint (Allaire *et al.* 2004). Therefore, only one analysis is required to compute the adjoint sensitivity.

The sensitivity in performance to a change in design was assessed here using the boundary method for design sensitivity described by Choi and Kim (2005). In adjoint variable structural sensitivity analysis, an adjoint equation is introduced by using the virtual displacement in the variational form of the governing equation as the adjoint variable. Evaluation of the design sensitivity requires solving for the real displacement and the adjoint variable. Typically the expressions for boundary sensitivity require a number of terms relating the problem to the boundary movement and external influences (e.g. external loading and restraints). For special cases such as strain energy minimization, where the boundary does not move in the region of an external influence the sensitivity of performance to the movement of the boundary reduces to

$$dJ = -\int_A \phi V_n dA, \quad (1)$$

where

$$\phi = \frac{\sigma^2}{2E} \quad (2)$$

is the strain energy density in response to a stress  $\sigma$  in a material of modulus  $E$ .  $\phi$  gives the sensitivity of strain energy to a unit outward boundary movement at the given point on the surface.  $A$  is the area of the boundary, and  $V_n$  is the design velocity, the outward movement of the boundary in the normal direction.

Eqn (1) assumes the shape sensitivity is first order, which is suitable for this work as the feature addition is assumed to cause only small changes in shape which vary smoothly everywhere except possibly in localized areas. This means that the domain of influence of second order design velocity, which is required for higher order approximations, is small. Further comment is made on this in the discussion.

Calculating the sensitivity for each new feature type allows an informed decision to be made about which should be added to give the greatest improvement in performance.

### **The objective of the work**

The objective of this work is to predict the change in performance caused by adding a small feature to the model using shape sensitivity analysis. The change in performance is predicted using adjoint analysis results, referred to here as sensitivity maps, and the assumed design velocity on the model boundary that adding the feature will cause.

This allows the optimum feature type and location to be predicted based on a single adjoint analysis of the model without any added features. Once the new optimal feature type and location is identified the feature added to the model, and the feature parameters will be used to allow it to grow using conventional shape optimization strategies.

It is assumed throughout that the area of the feature being added is infinitesimally small compared to the boundary area of the model into which it is being inserted.

## The model

Although the approach derived here is suitable for structural models of any shape, the problem considered in this paper is a cantilever beam, shown in Fig. 1, which has a point load applied at the top corner of the free end, and is bounded by four straight edges L1 to L4. The initial state of the beam was assumed to be a rectangular profile, with a constant thickness throughout.

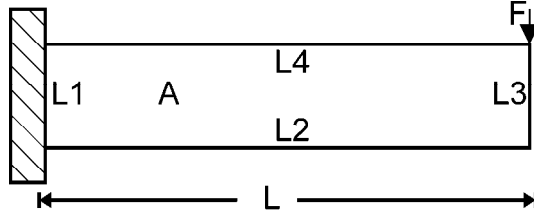


Fig. 1 The cantilever beam model

As will be described below, the design velocity due to adding stiffener, boss and pocket features is in the direction of the beam thickness, so the in-plane dimensions of the beam, including the length of each edge, are fixed throughout. The length of the beam is  $L$ , and the area of the beam boundary is  $A$ , and a constant volume constraint is imposed on the beam. This is implemented by specifying that the integral of design velocity over the boundary perpendicular to the plane must be zero, as in

$$\int_A V_n dA = 0. \quad (3)$$

In order to compare the effects of adding different features, the change in shape obtained by adding each feature type is normalized by enforcing the additional constraint that the root-mean squared design velocity is equal to the small value  $dV$ ,

$$\int_A V_n^2 dA = AdV^2. \quad (4)$$

## 3. A stiffener feature

The first feature type considered is the addition of a stiffener to one of the bounding edges of the model. A stiffener feature adds material along an edge to increase the stiffness of the structure. Here it is assumed the added feature will have a rectangular cross section. In the case of a cantilever beam, adding a stiffener to the top or bottom edge will create a beam with a T-shaped cross

section. The stiffener is shown in grey in Fig. 2. The parameters associated with the stiffener are the depth of the stiffener,  $\varepsilon$ , and the thickness of the stiffener, which is represented in the derivation as the design velocity over the area of the stiffener,  $V_I$ .  $V_0$  represents the design velocity over the area of the remainder of the beam, which is required to ensure there is no net change in volume.

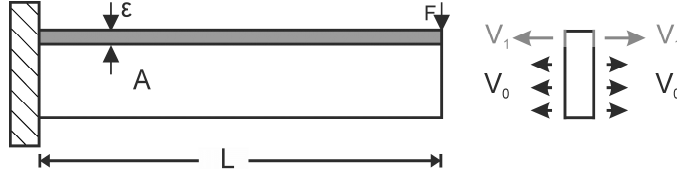


Fig. 2 Adding a stiffener

Enforcing the constant volume constraint for the above parameters yields

$$(A - \varepsilon L)V_0 + \varepsilon L V_I = 0. \quad (5)$$

Re-arranging for  $V_0$  and  $V_I$  gives

$$V_I = -\frac{(A - \varepsilon L)}{\varepsilon L} V_0 \quad (6)$$

and

$$V_0 = -\frac{\varepsilon L}{(A - \varepsilon L)} V_I. \quad (7)$$

Enforcing the unit 2-norm constraint for the above parameters yields

$$(A - \varepsilon L)V_0^2 + \varepsilon L V_I^2 = A dV^2. \quad (8)$$

By assuming  $\varepsilon L \ll A$ , and therefore  $A \approx A - \varepsilon L$ , and by substituting (6) and (7) into (8), and solving in the limit of small  $\varepsilon$ , results in

$$V_0 = -\sqrt{\frac{\varepsilon L}{A}} dV \quad (9)$$

and

$$V_I = \sqrt{\frac{A}{\varepsilon L}} dV. \quad (10)$$

Note that  $V_0$  is negative and  $V_I$  is positive, representing the fact that if the stiffener adds material to the beam, the remainder of the beam will have to become thinner to maintain a constant volume.



Using (1) the change in performance due to adding a stiffener of depth  $\varepsilon$  along an edge can be predicted using an analysis of the beam with no feature added, the sensitivity information along the edge to which the stiffener is to be added, and the assumed design velocity due to adding the stiffener feature  $V_I$  as

$$dJ = -\int_A \phi V_0 dA - \varepsilon \int_L \phi V_I dS. \quad (11)$$

Substituting  $V_0$  and  $V_I$  in (9) and (10) gives

$$dJ = \sqrt{\frac{\varepsilon L}{A}} dV \int_A \phi dA - \varepsilon \sqrt{\frac{A}{\varepsilon L}} dV \int_L \phi dS, \quad (12)$$

which simplifies to

$$\frac{dJ}{dV} = \sqrt{\varepsilon} \sqrt{AL} \left[ \frac{1}{A} \int_A \phi dA - \frac{1}{L} \int_L \phi dS \right]. \quad (13)$$

Eqn (13) allows the change in performance  $dJ$  to be predicted with respect to the depth of the stiffener  $\varepsilon$  and the magnitude of the overall shape change  $dV$ . For a 2D plane stress problem, the integrals in this equation represent the strain energy density integrated over the area and edge length respectively.

As might be expected, this demonstrates that the sensitivity to adding an edge stiffener will be large if the average strain energy density on the edge is much greater than the average over the associated face.

## 4. Boss and pocket features

The second and third feature types considered are boss (sometimes referred to as pads) and pocket features, which can be added to the interior of the domain. A boss is a lump of material of constant cross section which is added to the component, whereas a pocket feature is where a small cross section of material of a defined depth is removed from the component in the interior of the domain, but does not make a hole through the part. As for the stiffener the prediction of the change in performance due to adding these features is achieved using the analysis results for the component with no feature added, and the assumed design velocity due to adding the pocket of boss feature at different locations within the domain. The addition of both of these features is represented by Fig. 3. In a CAD model boss and pocket features are usually defined by a sketch of the profile, which is

extruded in the normal direction from the component boundary by a defined distance.

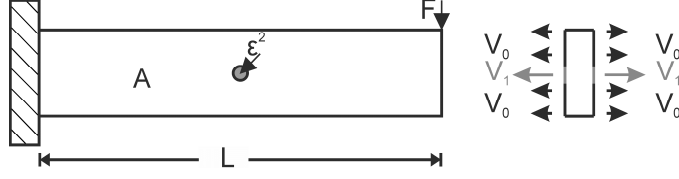


Fig. 3 Adding a boss or pocket feature

The parameters associated with these features are their cross sectional area,  $\varepsilon^2$ , and depth, which is represented in the derivation as the design velocity  $V_I$  over the area of the feature. The feature is not necessarily circular.  $V_0$  represents the design velocity over the surface area of the remainder of the beam, which is required to ensure there is no net change in volume.

The effect of adding a boss feature is considered first. Enforcing the constant volume constraint for the above parameters yields

$$(A - \varepsilon^2)V_0 + \varepsilon^2V_1 = 0. \quad (14)$$

Re-arranging for  $V_0$  and  $V_I$  gives

$$V_1 = -\frac{(A - \varepsilon^2)}{\varepsilon^2}V_0 \quad (15)$$

and

$$V_0 = -\frac{\varepsilon^2}{(A - \varepsilon^2)}V_1. \quad (16)$$

Enforcing the unit 2-norm constraint for the above parameters yields

$$(A - \varepsilon^2)V_0^2 + \varepsilon^2V_1^2 = AdV^2. \quad (17)$$

By assuming  $\varepsilon^2 \ll A$ , and therefore  $A \approx A - \varepsilon^2$ , and by substituting (15) and (16) into (17), and solving in the limit of small  $\varepsilon$ , results in

$$V_0 = -\frac{\varepsilon}{\sqrt{A}}dV \quad (18)$$

and

$$V_1 = \frac{\sqrt{A}}{\varepsilon}dV. \quad (19)$$

The sensitivity due to adding a boss can be calculated using an analysis of the beam with no feature present, the strain energy density at different points over the beam boundary, and the assumed design velocity due to adding the feature  $V_I$ , as

$$dJ = -\int_A \phi V_0 dA - \int_{\varepsilon^2} \phi V_I dA. \quad (20)$$

Substituting  $V_0$  and  $V_I$  in (18) and (19) gives

$$dJ = \frac{\varepsilon}{\sqrt{A}} dV \int_A \phi dA - \frac{\sqrt{A}}{\varepsilon} dV \int_{\varepsilon^2} \phi dA, \quad (21)$$

which simplifies to

$$\frac{dJ}{dV} = \varepsilon \sqrt{A} \left[ \frac{1}{A} \int_A \phi dA - \bar{\phi} \right] \quad (22)$$

where  $\bar{\phi}$  is the local value of strain energy density at the point where the boss is to be added. As might be expected, the sensitivity to adding a boss feature will be large if the strain energy density at the point where it is to be added is much greater than the average over the boundary of the model.

The derivation of the change in performance due to adding a pocket is the same as that of a boss, with the exception that the design velocity of the feature,  $V_I$ , is negative as it is inwards, and the design velocity of the remainder of the beam,  $V_0$ , is positive as it has to move outwards to conserve the overall volume. This leads to an expression for the sensitivity of adding a pocket feature as

$$\frac{dJ}{dV} = -\varepsilon \sqrt{A} \left[ \frac{1}{A} \int_A \phi dA - \bar{\phi} \right], \quad (23)$$

which is the negative of (22). The reverse is also true for location, and the most benefit will be obtained if the pocket is added to the point on the boundary where the strain energy density is small compared to the average over the boundary.

Note that in the derivation of sensitivity for a boss or a pocket feature nothing is being said about the cross sectional profile of the boss, rather only the cross sectional area is considered. This is explored further in section 6.

## 5. A hole feature

The next feature considered is the addition of a hole to the model, Fig. 4. A hole feature is typically added as a circle which is extruded through the part to remove

material. Whereas the stiffener, boss and pocket features are all examples of shape optimization, as their effect is to modify the shape of the boundary of the part, the insertion of a hole is an example of topology optimization because the topology of the model is changed.

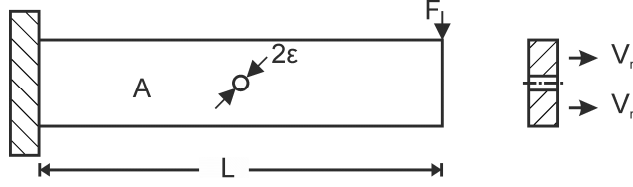


Fig. 4 Adding a hole

When the topology of the model is changed, such as by the addition of a hole feature, the concept of a topological derivative (Sokolowski and Zochowski 1999) is utilized. The topological derivative gives for each point on the domain the sensitivity of performance to adding an infinitesimal hole. Where strain energy is the objective function, the relevant topological derivative,  $D_T$ , for a model with Neumann boundary conditions on the hole (i.e. no surface traction), is

$$D_T = \frac{1}{2\tilde{E}} \left[ (\sigma_1 + \sigma_2)^2 + 2(\sigma_1 - \sigma_2)^2 \right]. \quad (24)$$

Here  $\sigma_1$  and  $\sigma_2$  are principal stresses, and  $\tilde{E}$  is the effective Young's Modulus

$$\tilde{E} = \frac{4k\mu}{k + \mu} \quad (25)$$

where  $k$  and  $\mu$  are the shear and bulk modulus respectively.

For a 2D model of thickness  $t$ , where the objective is to minimize strain energy, and where the boundary of the hole of radius  $\varepsilon$  is unloaded, the performance is

$$J_{hole,t=t} = J_{no\ hole,t=t} + \pi\varepsilon^2 t D_T. \quad (26)$$

To enforce the constant volume constraint, the thickness of the beam after the hole has been inserted,  $t'$ , will have to be larger than the thickness of the beam with no hole, which can be written

$$J_{hole,t=t'} = J_{hole,t=t} - \int_A \phi V_n dA. \quad (27)$$

A single expression governing the effect of adding a hole, under the constraint of constant volume, is generated by substituting (26) into (27)

$$J_{hole,t=t'} = J_{no\ hole,t=t} + \pi\varepsilon^2 t D_T - \int_A \phi V_n dA. \quad (28)$$

Enforcing the constant volume constraint requires that

$$(A - \pi\epsilon^2)t' = At, \quad (29)$$

meaning that the thickness of the beam after the hole is inserted is

$$t' = \frac{At}{A - \pi\epsilon^2}. \quad (30)$$

The design velocity of the face of the beam due to the addition of the hole is the change in thickness

$$V_n = t' - t. \quad (31)$$

Substituting (30) into (31) gives

$$V_n = \frac{\pi\epsilon^2 t}{A - \pi\epsilon^2}, \quad (32)$$

which expresses the design velocity in terms of the size of the hole and the thickness of the original beam. For this model all of the movement of the boundary occurs on the face of the beam, therefore  $V_n = dV$ .

Substituting (32) into (28) yields

$$J_{hole,t=t'} = J_{no\ hole,t=t} + \pi\epsilon^2 t D_T - \int_A \phi dA \frac{\pi\epsilon^2 t}{A - \pi\epsilon^2}. \quad (33)$$

For small values of  $\epsilon$ , where  $A - \pi\epsilon^2 \approx A$ , (33) becomes

$$J_{hole,t=t'} = J_{no\ hole,t=t} + \pi\epsilon^2 t \left( D_T - \frac{\int_A \phi dA}{A} \right). \quad (34)$$

Therefore the change in performance due to adding the hole feature is

$$dJ = \pi\epsilon^2 t \left( D_T - \frac{\int_A \phi dA}{A} \right). \quad (35)$$

## 6. Selecting the optimum feature location and type

In sections 3 to 5 equations have been derived which allow the change in objective function due to adding different feature types to a model to be predicted based on an analysis of the model without the feature present, and assuming the design velocity adding the feature will cause. When the model has been

optimized using its current parameter values, these equations give a rationale for determining the optimum feature to add to a model in order to obtain the greatest improvement in performance. Once added the parameters which define the new features can also be used as design variables.

In this section the addition of each of the features is compared for the cantilever beam model. The predicted improvement in performance for each is also compared to the results of a finite element analysis (FEA) to determine the accuracy of the derived equations. The beam analysis used in this paper is included to allow the effects of adding different parameters to be compared. As the model was used for comparison purposes a Young's modulus ( $E$ ) of  $1\text{N/m}^2$  and Poisson's ratio ( $\nu$ ) of 0.3 were used. The length, height and thickness were 10m, 2m and 1m respectively. The load applied was 1N.

A 2D linear elastic representation of the beam was meshed using quadratic triangular elements to obtain the strain energy density information for the beam. Due to the fact the initial analysis model contained no small features, a reasonably coarse mesh was used, Fig. 5.

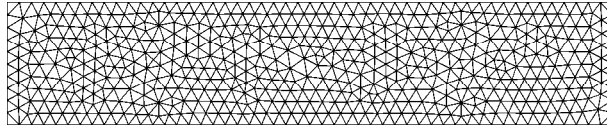


Fig. 5 Meshed beam

The strain energy density distribution through the beam is shown in Fig. 6, and only these results were used to predict the performance after each feature was added. It should be noted that there is a localized region of high strain energy density located at the top right hand corner corresponding with where the load was applied.

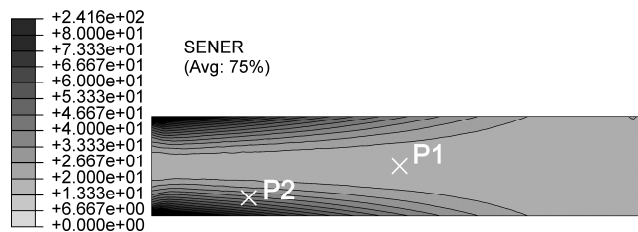


Fig. 6 Strain energy density in the beam

## Change in performance due to adding a stiffener

Considering the cantilever beam in Fig. 1, Fig. 7 shows the strain energy density integrated over each edge in the model. L2 and L4 have large values compared to L1 and L3, indicating that adding a stiffener to either of these edges will result in the greatest improvement in performance. This provides a rational approach for deciding where to add a new feature, and can be computed before any modification is carried out on the CAD model.

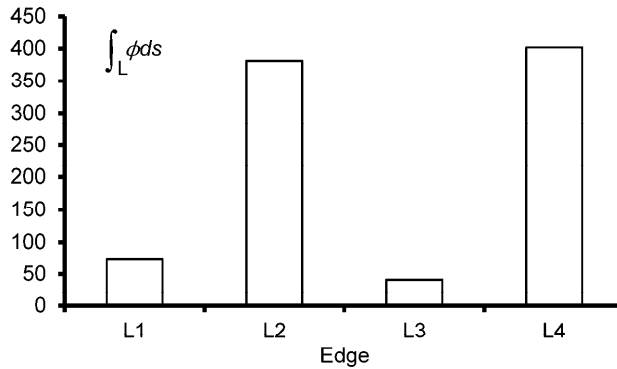


Fig. 7 Strain energy density integrated over each edge as labeled in Fig. 1

Fig. 8 compares the change in performance predicted for the addition of a stiffener to the top edge, L4. The  $J(\text{Predicted})$  values were computed using (13), where the strain energy density in the model was computed by a finite element analysis of the beam without a stiffener feature, and by assuming the design velocity caused by adding features of different size.  $J(\text{FEA})$  was computed for validation by finite element analyses of the beam geometry, with a stiffener feature added to the top edge, and the values of  $\varepsilon$  and  $V_n$  varied for each analysis. In all of the finite element models with a stiffener, the overall movement  $dV$ , was the same at 0.05.

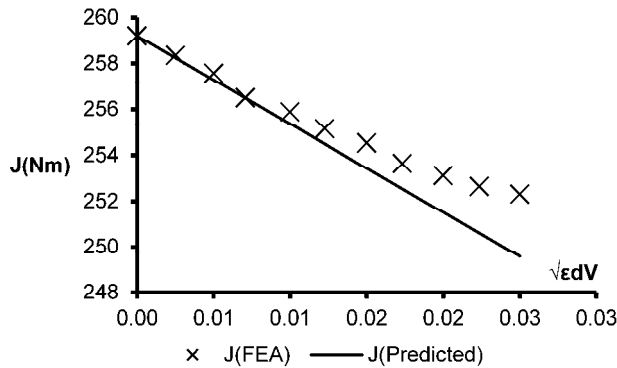


Fig. 8 Comparison of performance due to adding a stiffener feature

## Change in performance due to adding a boss or pocket

In the derivation of sensitivity due to adding a boss or pocket feature it was pointed out that for features of the same area and offset from the face, the sensitivity of one is the negative of the other. For the small feature size being considered, the shape of the boss or pocket does not have an effect on the performance. Feature shape will begin to have an effect as the feature begins to grow in later optimization stages. Eqn (22) suggests that the most appropriate location to add a small boss feature is where the strain energy density is highest within the domain. Eqn (23) suggests that the most appropriate location to add a small pocket feature is where the strain energy density is lowest within the domain.

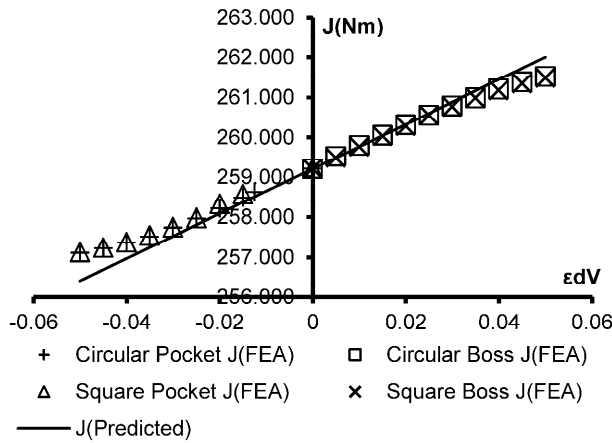


Fig. 9 The performance after adding boss and pocket features of different profile

Fig. 9 compares the change in performance predicted for the addition of a boss and pocket respectively to the neutral axis of the beam halfway along its length (the location shown in Fig. 3 [Error! Reference source not found.](#)) when  $dV=0.05$ . Note that this location was not selected because it was the optimum for either feature, as suggested by (22) and (23), but to allow the effect of adding a boss or pocket feature on performance to be contrasted. It should be noted that when modeling a pocket, a very small  $\epsilon$  value results in a  $V_I$  value greater than the thickness of the beam. Such features have not been included in Fig. 9 as they are hole features as opposed to a pockets.

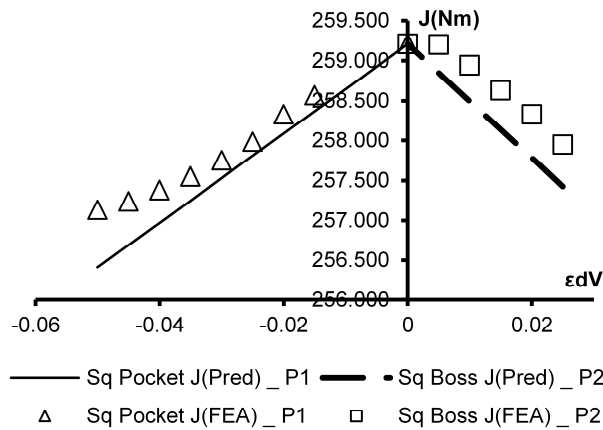
(22) and (23) predict the change in performance due to adding a boss or pocket feature to a CAD model, but the profile of the boss is simply represented as  $\epsilon^2$ .

Fig. 9 compares the performance when circular and square bosses and pockets are added to the center point of the model and  $dV=0.05$ . In Fig. 9 J(Predicted) was



computed using (22) and (23) with the strain energy density computed by a finite element analysis of the beam without a boss or pocket feature, and the assumed design velocity which will result from adding the pocket or boss feature.  $J(\text{FEA})$  was computed for validation using finite element analyses of the beam geometries, with the feature added to the same location. The values of  $\varepsilon$  and  $V_n$  were varied for each analysis, and controlled by  $dV$ .

It is apparent from the graph that pockets and bosses have the opposite effect on performance, as predicted by the equations. Finite element analysis has confirmed that small boss and pocket features of circular and square profiles produce the same change in objective function for the same small feature area. It is anticipated that this will be true for all profile shapes of the same area, providing the lateral dimensions of the profile are similar. If this is not the case, a framework for establishing the sensitivity to more complex features like long slender pockets or pads has been identified by Gopalakrishnan and Suresh (2008). A second analysis was carried out where the effect on performance of the addition of two features to the model was compared. The feature locations are at crosses P1 and P2 shown in Fig. 6. P1 is at the centre of the beam, in an area of low strain energy density, and therefore (23) suggests that a pocket feature should be added to improve performance. P2 is in an area of high strain energy density, and therefore (22) suggests that a boss feature should be added at this location to improve performance. The effect of each feature is shown in Fig. 10.



**Fig. 10 Adding features to different locations in the model**

Fig. 10 demonstrates that adding a pocket to position P1 will cause a greater improvement in performance than adding a boss to position P2. It is also worth noting that due to the position of P2 that the maximum feature size possible will

be reduced due to the close proximity to the model boundary. This will limit the benefit this feature will contribute in future optimization.

### Change in performance due to adding a hole

Hole features are similar to the bubble features described by Eschenauer *et al.* (1994). Here only circular hole features are considered, but Eschenauer *et al.* (1994), Chua *et al.* (2010) and Weiss (2010) provide approaches which allow this circular hole feature to take on a new shape during optimization. From (35) the best location for adding a hole feature is where the strain energy, stresses and  $D_T$  are small compared to the average over the domain. This can be determined from the single analysis of a beam with no holes (Wang and Wei (2005)). Garcia and Steven (2004) suggest that the optimum location for a hole is where the stress in the domain is at its minimum. In their conclusions Eschenauer *et al.* list determining the position for the bubble features as future work, although they do note that the optimal position of a bubble within the structure corresponds to the point of minimal value of the “characteristic function”. The characteristic function is introduced by Eschenauer *et al.* as an objective function with integral functionals as constraints.

Fig. 11 compares the change in performance predicted for the addition of an infinitesimal hole to the center of the beam with beams containing a small finite hole. The location was selected to allow comparison with the boss and pocket features.  $J(\text{Predicted})$  was computed using (35), and the strain energy density in the model computed by a finite element analysis of the beam without a hole feature.  $J(\text{FEA})$  was computed by finite element analyses of the beam geometries with a hole feature added, and the value of  $\varepsilon$  varied for each analysis. Eqn (35) suggests that the topological sensitivity to the addition of a hole is greatest where the strain energy is small compared to the average.

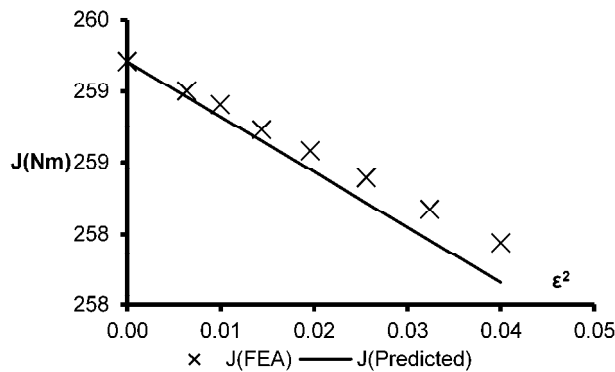


Fig. 11 Comparison of performance due to adding a hole feature

## 7. Optimization based on CAD features

In this paper an iterative approach is proposed for generating feature-based CAD models with sufficient flexibility to allow highly optimized components to evolve based on feedback from CAE simulation results. The proposed approach, Fig. 12, is first to shape optimize the model using the parameters belonging to the existing CAD features in the model. When these are at their optimum values, the techniques described in this paper to derive sensitivity to adding infinitesimal features could be used to add new small finite CAD features at the optimum location, followed by a further shape optimization using the existing parameters plus the parameters associated with the new features. This should ensure that the shape flexibility provided by the new feature parameters is added to the model where the current parameterization is not capable of moving the boundary in the optimum fashion.

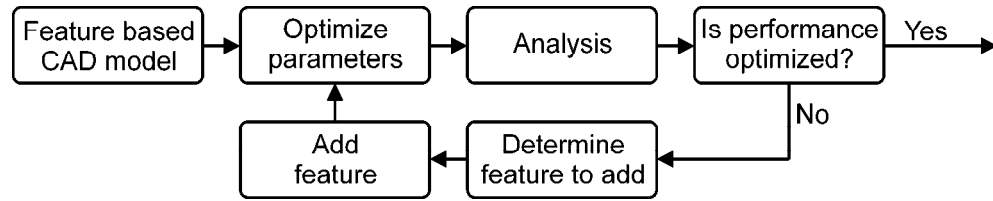


Fig. 12 Suggested optimization procedure

The cycle of feature addition to an existing optimized shape could then be repeated. Adding a new feature before optimizing using the current parameterization, could result in a new feature being inserted where the current parameterization already provides shape movement.

Fig. 8 to Fig. 11 compare the performance of the model after new features have been added. It is clear from the analyses that adding a stiffener to edge L4 has the greatest potential to reduce the objective function when compared to the boss, pocket or hole feature added to the center of the beam. This provides a clear metric by which to choose which feature to add, although for the case in question it should be noted that the stiffener feature was positioned at its optimum location, whereas the boss, pocket and hole features were inserted at a given location. Inserting these features at their optimum location, as described above, may produce a better improvement in performance.

Interestingly, the local stress concentrations under the point load in Figure 6 suggest that a boss or local thickening is required where the load is applied.

It is anticipated that a number of new features will have to be added to optimize a component, and it is proposed the existing parameters should be set to their optimum values before a new feature is added. Any addition of complexity of the model is based on an analysis of incremental benefits. When the optimum shape has been obtained the sensitivity map will be constant over the component boundary (Pederson 1992). When this point has been reached, a change in topology of the model is required.

It is also anticipated that the greatest benefit will be obtained by combining the addition of the features discussed here with the added flexibility obtained by changing the boundary edges from straight lines to spline curves. This can be considered another type of feature addition, and the shape can then be optimized based on the positions of the control points as shown by Eschenauer *et al.* (1994), Chua *et al.* (2010) and Weiss (2010).

## 8. Discussion

Equations have been derived which allow the sensitivities of adding a stiffener, a boss, a pocket and a hole feature to be predicted. Fig. 8 to Fig. 11 show that the difference between the predicted performance and the performance computed using FEA is very small for all feature types, especially at small feature sizes. This suggests that the equations can be used with confidence to calculate the sensitivity to adding one of these features, when the feature size is small. It is also shown that the smaller the dimensions of the feature being added the more accurate the prediction, which is expected for this type of shape sensitivity analysis approach where a first order variation in performance is assumed. The first order shape variation is valid for cases where the feature is large laterally compare to the feature depth.

The approach presented is efficient because the effect of adding the different features can be evaluated using the results of just one adjoint analysis, without the need to use computational analysis to determine the effect of adding a feature. The ideas have been tested here based on strain energy, which is a simple, self-adjoint problem, for which computational analysis is relatively cheap. It is envisaged the greatest benefit will probably be for large CFD problems, where the costs of running an analysis to determine the effect of adding features are prohibitive. The philosophy proposed here is optimization by incrementally

adding complexity to an existing simple representation, which mimics the natural design process.

When adding the features discussed here the design velocity around the boundary of the feature is not continuous, Fig. 13. This will cause stress concentrations to occur around the feature boundary and errors in sensitivity prediction due to second and higher order variations in design velocity. If however the feature is a shallow one, so that the length of the perimeter times the radius of influence of any stress concentration due to the discontinuity of design velocity is small compared to the area of the feature, the errors in computing feature sensitivity will be negligible. Though this has not been tested, the prediction of feature sensitivity for long thin rectangular bosses or pockets would probably be less successful.

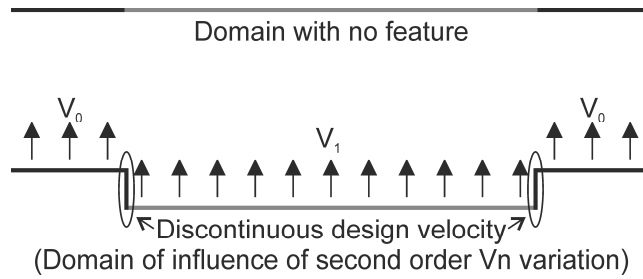


Fig. 13 Discontinuous design velocity at feature boundary

The approach described here allows the different options for feature addition to be compared in terms of performance improvement obtained, but says nothing about the effect on other performance measures, or the effect on manufacturing cost. For example in manufacturing terms a round hole in a constant thickness beam is likely to be cheaper than manufacturing a beam with a boss on it, but this is not accounted for in the approach described above. This may suggest a penalty should be added to different feature types for optimization depending on predicted manufacturing complexity, or that an engineer who is capable of intelligently interpreting the performance improvement predicted should use the sensitivities to guide design decisions.

When the optimum feature type to add, and location to add the feature have been identified, a simple script can be used to introduce a small finite feature into the CAD model. Once this feature has been added, a new mesh will need to be created for the model. The CAD models produced should allow the parameters in the CAD model to be used as optimization variables, while providing sufficient flexibility in the shape for a highly optimized model to be created. The process

used to optimize the component using the added parameters has not been commented on and is an obvious area for future work.

## 9. Conclusions

- The sensitivity of performance to the addition of infinitesimal stiffener, boss, pocket and hole features in a 2D domain have been derived and compared. These sensitivities depend on the adjoint sensitivity map over the domain boundary and the design velocity (shape change) induced by adding an infinitesimal feature
- The variation in adjoint sensitivity over the faces or edges of a CAD model can be used to identify where new features should be added
- An incremental approach to adding features to a CAD model for use in optimization has been described
- The approach described here has evaluated the sensitivity of adding four features to a model. The approach could be extended to evaluate the sensitivities of adding other conventional CAD feature types.

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